Advertising, Consumption and Asset Prices

Emilio Bisetti*

May 1, 2016

Abstract

One of the core predictions of dynamic asset pricing theory is that expected consumption and equity returns should be correlated, yet this prediction finds little empirical support. In this paper, I show that advertising expenditure growth is a robust predictor of consumption growth in aggregate post-war US data. Therefore, it also predicts excess returns on equity. I build a dynamic stochastic general equilibrium model of frictions in the goods market to explain these empirical findings. In the model, firms invest in advertising expenditures to attract new customers and build long-lasting customer relationships. This customer capital determines future consumption of households. A calibrated version of the model is able to replicate the predictive power of advertising on consumption growth and equity returns that is observed in the data. Frictions in the goods market are a key component of the model in generating these predictability patterns.

*Tepper School, Carnegie Mellon University. Email: bisetti@cmu.edu. I would like to thank, without implicating them, Burton Hollifield, Nicolas Petrosky-Nadeau, Chris Telmer, Ariel Zetlin-Jones, as well as seminar participants at Tepper for comments, insights and suggestions.

1 Introduction

The post-war period has seen a steady increase in aggregate advertising, and a dramatic evolution in the way companies use advertising to induce the purchase of their products. The introduction of new means of communication such as television and the internet has been quickly followed by the effort of companies to use these new means to inform potential customers about their products. Despite the existence of an entire field of economics studying how advertising can influence consumption choices (Bagwell (2007)), and despite the central role that consumption plays in modern financial economics, surprisingly little research has however analyzed the implications for financial economics of the advertising-consumption relation. This paper aims to be the first to explore these implications, both empirically and theoretically, through the lens of consumption-based asset pricing.

I begin by documenting an empirical relationship between aggregate advertising expenditures, consumption and equity returns in the United States. I first show that aggregate advertising growth predicts future aggregate consumption growth at annual horizons of one to two years. This predictability relation is time-varying and holds across different robustness tests in post-war data. Then, I show that advertising and consumption growth together predict excess returns, and they do so better than most predictors such as the dividend-price ratio and the dividend payout ratio. In particular, high advertising growth predicts high future returns, and high consumption growth predicts low future returns.

I build a model of frictional search in the goods market to replicate the predictability found in the data. The model features two goods, one of which is exogenously endowed to households. The second good is sold by firms on a goods market characterized by two frictions. The first friction is an informational friction such that, absent advertising, households are only aware of the existence of their endowment. Firms use advertising to overcome this friction and search for new customers among the households. Once a firm attracts a household, the firm and the household form a customer relationship that lasts for multiple (as in Gourio and Rudanko (2014)). The second friction is an advertsing externality that makes the customer search process more difficult for each firm whenever advertising by other firms is high. Following the labor search literature, I call this externality a goods market congestion effect. The model has direct implications for the impact of advertising and customer relationships on household consumption and equity returns. On the household side, advertising shifts consumption away from the endowment good and creates a persistent component in the consumption of goods produced by firms. On the firm side, customers are risky assets. In bad times, firms may want to decrease their stock of customers but they are prevented from doing so because their advertising cannot be negative. Conversely, in good times firms would like to increase their customers, but because advertising by other firms is also high the congestion effect makes their advertising less effective in attracting new customers.

The model is able to replicate the predictive power of advertising growth and consumption growth on equity returns, as advertising growth induces negative co-variation between expected marginal utility and equity returns. High advertising growth shifts expected consumption away from the numeraire and therefore increases the numeraire's expected marginal utility. At the same time, high advertising growth lowers expected returns from advertising, which in the model are paid in units of the endowment good. This happens because i) high growth in advertising reduces the future marginal revenues of the firm and ii) high advertising growth at the individual firm level generates high aggregate advertising growth which reduces the likelihood for individual firms to attract new customers. Put together, these conditions imply that times when advertising growth is high are times of low expected returns, high expected marginal utility and high returns. Finally, the results show that the goods market congestion effect is a key element in driving the predictive power of advertising growth on excess returns. To compensate the externalities arising from advertising individual firms widely vary their advertising decisions depending on the state of the economy. The model therefore generates large shifts in advertising growth that map into large shifts in the growth of consumption, marginal utility and marginal profits, and that drive the predictive power of advertising on future consumption and returns. A counterfactual exercise shows that, absent any goods market congestion, the predictive power of advertising growth on excess returns vanishes.

Related Literature The contribution of the paper to the literature is twofold. First, the paper provides empirical evidence supporting the idea that standard consumption-based asset pricing models hold when conditioning on variables that provide information about agents' future expectations (Campbell and Cochrane (2000)). Different from the previous literature, which conditions on variables that contain a price and therefore directly predict future expected returns (Ferson and Schadt (1996), Jagannathan and Wang (1996), Cochrane (1996) and Lettau and Ludvigson (2001b)), I however predict returns using advertising through the channel of future expected consumption. My results are in this sense close to those in Savov

(2011).[1] Second, this is the first paper to explore the theoretical implications of advertising, goods market frictions and customer capital for aggregate consumption and asset pricing. In this respect, my work relates to two strands of literature. From the macroeconomics standpoint, frictions in the goods market have been recently shown to be a key ingredient in generating features observed in business cycles. Petrosky-Nadeau and Wasmer (2015) demonstrate goods market frictions as an intuitive way to endogenously generate persistent business cycle fluctuations. In a similar spirit, Den Haan (2013) analyzes the role of inventories as coming from imperfect market clearing in generating business cycles, while Storesletten, Rull, and Bai (2011) show that goods-market frictions allow a model with demand shocks to match most of the features of a standard model with productivity shocks. Finally, Hall (2014) relates the pro-cyclical variation of advertising expenditures to macroeconomic wedges, and in particular to frictions in the goods market. From the financial economics standpoint, my work builds on two recent sub-fields of the production-based asset pricing literature (Cochrane (1991, 1996) and Jermann (1998)). The first builds on Berk, Green, and Naik (1999) to analyze the impact of growth options in intangible capital (Ai, Croce, and Li (2013)), organization capital (Eisfeldt and Papanikolaou (2013)) and brand capital (Belo, Lin, and Vitorino (2014) and Vitorino (2014)) on the cross-section of expected stock returns. The second focuses on how search frictions in the labor market affect asset prices (Kuehn, Petrosky-Nadeau, and Zhang (2012), Belo, Lin, and Bazdresch (2014) and Kuehn, Simutin, and Wang (2014)). Finally, from a modeling point of view the two papers most closely related to mine are Drozd and Nosal (2012) and Gourio and Rudanko (2014), which however respectively focus on international prices and the cross section of firm characteristics.

2 Aggregate Advertising Expenditures and Equity Returns

Robert J. Coen from the advertising company Erickson-McCann used to regularly publish data on aggregate advertisement expenditures in the United States. The dataset ranges from 1900 to 2007 and includes, among other variables, U.S. aggregate expenditures for advertising on newspapers, periodicals, yellow pages, radio, television and internet.[2] In Figure 1, I explore the time-series evolution and composition of post-

[1] He uses garbage as a measure of realized consumption to test the consumption-based asset pricing model, while I use advertising as a measure of expected consumption.

[2] The data can be found on Douglas Galbi's website: http://purplemotes.net/2008/09/14/us-advertising-expenditure-data/. For the years 2007 to 2010, Hall (2014) updates this dataset using revenue data from companies in the information sector published by the Census Bureau. These data are no longer available, but can be found on Hall's website.

war advertising by breaking the variable in two broad categories, physical and non-physical advertising. I define physical advertising as the sum of advertising on newspapers, periodicals, magazines, direct mail, yellow pages, farm publications, billboards and business papers, and non-physical advertising as the sum of advertising on radio, television, and internet. The Figure shows that the level of aggregate advertising expenditures in the US is five times larger in the late 2000s than in the 1950s, and that advertising growth is mainly due to physical advertising growth. Second, traditional physical advertising and modern non-physical advertising are complements rather than substitutes. Despite the advent of television and internet advertising and the increasing relative importance of these channels (Panel B), the average U.S. company in 2010 still spends more than twice as much in physical than in non-physical advertising.

[Figure 1 about here]

In Figure 2, Panel A, I compare the post-war evolution of per-capita advertising expenditures and per-capita consumption in the United States. The data for consumption come from personal consumption expenditures in the NIPA tables, and both advertising expenditures and consumption are expressed in 2005 US dollars, using the Consumer Price Index (CPI) for consumption and the Producer Price Index (PPI) for advertising expenditures.[3] As extensively documented in the literature (see Hall (2014) and references therein), advertising is a pro-cyclical variable, and therefore highly correlated (but not cointegrated, see Appendix A) with consumption. In Panel B I plot the advertising-consumption ratio. The Figure shows that the ratio is a slowly-moving process, decreasing during recessions in the late 80s and early 2000s and expansions in the 50s and 90s expansions.

[Figure 2 about here]

In Figure 3, I finally plot advertising growth, consumption growth and excess returns on U.S. equity in the post-war period. The data for excess returns, defined as the yearly returns on the S&P 500 minus the one-year interest rate, come from Robert Shiller's website. Panel A of the Figure shows that the growth rates in advertising and consumption are highly correlated, advertising growth is more volatile than consumption growth and (especially after 1980) leads consumption growth. Panel B similarly shows a positive correlation between advertising growth and excess returns on equity.

[3] I keep this definition of consumption through the rest of the paper. The main results of the paper hold when I use more granular definitions of consumption such as consumption of nondurable goods, durable goods and services.

[Figure 3 about here]

2.1 Consumption Growth and Excess Returns Predictability

In this Subsection I show that advertising expenditures growth predicts consumption growth at predictive horizons of one to two years, and that advertising and consumption growth jointly predict excess returns at horizons of one to four years. Table 1 shows summary statistics for advertising expenditures growth, consumption growth and other known predictors.

Advertising growth (Δa) has a mean of 2.3 percent and a standard deviation of 5.6 percent, respectively three times higher than consumption growth. The variable is positively correlated with the dividend-price ratio, the earnings-price ratio and the payout ratio, so that times when corporate earnings are high are also times when advertising expenditures are high. Moreover, Δa is positively correlated with the Lettau and Ludvigson (2001a) cointegrating residual *cay*, so that advertising expenditures grow whenever whenever consumption is above its long-run equilibrium level. Finally, consumption and advertising growth are mildly autocorrelated with AR(1) coefficients of 0.27 and 0.39 (t-statistics of 2.04 and 3.04), respectively, but the null hypothesis of a unit root in augmented Dickey and Fuller (1979) tests is rejected for these two time series (the p-values of the tests are equal to zero up to four decimal points).

[Table 1 about here]

Table 2 presents the main empirical results of the paper, the predictive power of advertising growth on consumption growth and the predictive power of advertising and consumption growth on excess returns on equity. Panel A reports coefficient estimates and associated Hansen and Hodrick (1980) $t-$statistics for predictive regressions of cumulative consumption growth from year t to year $t+\tau$ ($\Delta c_{t \to t+\tau}$), using lagged consumption growth and lagged advertising growth as predictors. In the Table, the predictive horizon τ varies from one to four years. Specification (2) shows that lagged consumption growth predicts future consumption growth only up to one year in the future. Specifications (1) and (3) show that advertising expenditures predict consumption growth at horizons of one and two years. In particular, specification (3) shows that the predictive power of current consumption growth in forecasting future consumption growth in specification (2) arises from the component of consumption growth correlated to advertising growth.[4]

[4] As an additional experiment, I regress consumption growth on a constant and advertising growth, and use the resulting

Section 2.2 and Appendix A provide additional robustness tests for the predictive power of consumption growth on advertising growth. Panel B of Table 2 similarly reports the coefficient estimates and t-statistics for predictive regressions of cumulative excess returns ($r^x_{t\to t+\tau}$), using the same predictors as in Panel A. Specifications (1) to (3) show that even if consumption growth and advertising growth do not predict excess returns individually (but consumption at long horizons), together they predict excess returns at any horizon from one to four years. In particular, conditional on consumption growth high current advertising growth predicts high future excess returns.

[Table 2 about here]

In Table 3, I compare the predictive power of advertising and consumption growth to the predictive power of the predictors summarized in Table 1. As the previous literature documents, the dividend-price price-earnings ratios are effective at predicting long-horizon returns, while the predictive power of the payout ratio and term spread decreases with the predictive horizon. Advertising and consumption growth, similar to *cay*, have high predictive power at any predictive horizon. The term spread is the only variable that has stronger predictive power (as measured by the predictive regression's R-squared) than advertising and consumption at any horizon, while *cay* has higher predictive power at horizons of three and four years.

[Table 3 about here]

In Table 4, I run tri-variate excess returns predictive regressions using advertising growth, consumption growth and one of the other predictors as regressors, in the spirit of Huang (2015). For every predictive horizon I consider, advertising growth (and consumption growth, omitted in the Table) is always a significant predictor of excess stock returns. The only variables that are jointly statistically significant with advertising and consumption growth are the term spread and the payout ratio. For horizons of two years, the default spread, inflation and *cay* are jointly significant. Finally, at horizons of three and four years, *cay* is the only jointly significant predictor of excess returns.

[Table 4 about here]

residual to predict cumulative consumption growth. The null hypothesis of no predictive power of the residual cannot be rejected for any horizon from one to four years (p-values of 0.996, 0.448, 0.423 and 0.457, respectively).

2.2 Robustness

Table 5 shows the results of the estimation of a Vector-Autoregressive model of order two for advertising and consumption growth. The estimation results show that advertising growth is predicted by consumption growth and is autocorrelated conditional on advertising growth. On the other hand, consumption growth is not conditionally autocorrelated and is predicted by advertising expenditures growth.

[Table 5 about here]

Table 6 shows the results of Table 5 for different sub-samples of my dataset. In Panel A, I report the results for the 1922-2009 sample, while in Panel B for the 1982-2009 sample.[5] The results show that the dynamics of the advertising-consumption relationship dynamic dramatically change over the course of the last century. The sign of the VAR coefficients does not change across different samples, but their magnitude and statistical significance increases when I restrict the sample to more recent years. The time-varying relation between advertising growth and consumption growth therefore limits the use of advertising growth as an instrumental variable for expected consumption growth, and rathers calls for a model to explore the joint dynamics of the variables.

[Table 6 about here]

In Table 7, I test the robustness of advertising growth in predicting consumption growth, by augmenting specification (3) of Table 2 with an additional predictor. To the best of my knowledge, the term structure of the interest rates is one of the few variables known to predict consumption growth at long horizons. I follow Harvey (1988) and run consumption growth predictive regressions using the short-term risk-free rate and the term spread as a proxy for the term structure. Specification (1) of Table 7 confirms that the term structure is a good predictor of consumption growth at horizons from one to three years. Specification (2) shows that the predictive power of advertising growth significantly decreases the predictive power of the term structure at any predictive horizon.

[Table 7 about here]

[5] Tests for cointegration between consumption and advertising in these two samples fail to reject the null hypothesis of no cointegration.

Figures 4 and Table 8 report the results of my third robustness test, which measures the out-of-sample performance of advertising growth in predicting excess returns using both moving-window and expanding-window regressions. In moving-window regressions, I use a fixed rolling window of fifty observations as fitting sample. In expanding-window regressions, I start with fitting the model on the first fifty annual observations, and expand the sample one observation at a time to obtain the full 1950-2010 sample. Figure 4 plots point estimates for the coefficients of one-year-ahead excess returns predictive regressions, as well as their associated 95 percent confidence intervals, for the out-of-sample period 1980-2010. In both moving regressions (Panel A) and expanding regressions (Panel B), the regression coefficient associated to advertising growth is around one (but for years 2003-2008 in moving regressions), and always statistically different from zero at a 95 percent confidence level.

[Figure 4 about here]

Finally, Table 8 reports the results of the out-of-sample R-squared statistics of Campbell and Thompson (2008) and the adjusted mean squared prediction error (MSPE) statistics of Clark and West (2007). The out-of-sample R-squared displays similar values in moving and expanding regressions. The within-sample R-squared increases with the predictive horizon. The adjusted-MSPE statistics speaks in favor of long-run predictability. The (one-sided) t-statistic for a difference in predictive accuracy between within sample and out-of-sample predictions is rejected in both moving and expanding regressions at a five percent confidence level for horizons of two to four years.

[Table 8 about here]

Finally, in Appendix B I show that advertising growth predicts a component of aggregate consumption growth not captured by the Bansal and Yaron (2004) long-run risk.

In the next Section, I introduce a dynamic investment-based asset pricing model of frictions in the goods market to replicate the observed predictive power of advertising expenditures on aggregate consumption growth and excess returns.

3 Model

The model is a discrete-time, dynamic stochastic general equilibrium model with two goods. The economy is populated by a continuum of identical households and a continuum of identical firms. The model features two key frictions. First, absent advertising households in the economy are only aware of the existence of their endowment (numeraire) good. Firms overcome this friction by spending resources to advertise their product. Once firms and customers match with each other, they form a relationship that lasts for multiple periods. Second, the advertising process is subject to search externalities. Times when all the firms in the economy post many advertisements are also times when it is harder for an individual firm to attract a customer by posting an advertisement (ad). In particular, once every firm has posted its ad's, the sum of the individual ad's in the economy determines the probability that one ad will turn into a customer for an individual firm.[6] I denote this probability by λ. I assume that product search is costless for the household, and normalize the household search cost to one. Using a den Haan, Ramey, and Watson (2000) function with elasticity $\vartheta > 0$, the matching function between a household and an ad is given by

$$G\left(ad\right) = \frac{ad}{\left(1 + ad^{\vartheta}\right)^{1/\vartheta}}. \tag{1}$$

Denoting aggregate variables with uppercase letters, the probability λ that an advertisement attracts a household is a function of the total ad's in the economy:

$$\lambda\left(AD\right) = \frac{G\left(AD\right)}{AD} = \frac{1}{\left(1 + AD^{\vartheta}\right)^{1/\vartheta}}. \tag{2}$$

Finally, customer relations are long-lasting. I denote the stock of firm customers (customer capital, Gourio and Rudanko (2014)) by n and assume that in each period t the firm loses an exogenous fraction $\varphi \in [0, 1]$ of its customers. The aggregate law of motion for customer capital between period t and period $t+1$ is then

$$N_{t+1} = \left(1 - \varphi\right) N_t + G\left(AD_t\right) \tag{3}$$

$$= \left(1 - \varphi\right) N_t + \lambda\left(AD_t\right) AD_t. \tag{4}$$

[6] For tractability, I abstract from the difference between customer and marketing/brand capital (Drozd and Nosal (2012)), where advertising expenditures build marketing capital, which in turn determines the likelihood of attracting new customers.

3.1 Firm Problem and Return on Equity

Firms enter each time period with a stock of customers, observe the aggregate endowment of the numeraire good and decide how much to spend in advertising to build their customer capital. Firm revenue is the product between the unit price of the manufactured good and the number of firm customers. Moreover, firms pay a convex advertising cost to attract customers. Three assumptions on firm profits allow to simplify the analysis while retaining the model's main insights. First, firms extract all the matching surplus from households, so that the manufactured good's price is the marginal rate of intratemporal substitution between the manufactured good and the numeraire. This eliminates the issue of time-inconsistent pricing (Nakamura and Steinsson (2011)). Second, I do not explicitly model production of the advertised good. I assume that the advertised good is always available for firms to buy and re-sell to households at the price of purchase once a firm finds a customer. This allows to reduce the state space and focus on the implications of advertising externalities for return predictability. Finally, the model features convex advertising costs to reduce the volatility of advertising.

As in Liu, Whited, and Zhang (2009) and Kuehn, Petrosky-Nadeau, and Zhang (2012), the firm's problem at time t is to maximize the discounted expected value of its dividend stream S_t, subject to the law of motion for its customer base and a non-negativity constraint on advertising. Let P_t denote the period-t relative price of the advertised good in terms of the numeraire. The representative firm's problem is

$$S_t = \max_{\{AD_{t+j}\}_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} M_{t+j} \left[P_{t+j} - \frac{\chi}{2} \left(\frac{AD_{t+j}}{N_{t+j}} \right)^2 \right] N_{t+j}, \tag{5}$$

subject to, for all j,

$$AD_{t+j} \geq 0 \tag{6}$$

and the law of motion (3). Here, M_{t+j} denotes the stochastic discount factor (SDF) between t and $t+j$, χ is a convex adjustment cost parameter and (6) is a non-negativity constraint on effort. Since the matching probability λ_t is greater than zero, (6) can be re-written as

$$\lambda_{t+j} AD_{t+j} \geq 0. \tag{7}$$

Substituting the first constraint into (5), and respectively denoting by μ_t^n and μ_t^λ the time-t Lagrange multipliers on (3) and (7), the problem's first order conditions are

$$\mu_t^n = \frac{\chi}{\lambda_t} \frac{AD_t}{N_t} - \mu_t^\lambda, \tag{8}$$

$$\mu_t^n = \mathbb{E}_t M_{t+1} \left[P_{t+1} + \frac{\chi}{2} \left(\frac{AD_{t+1}}{N_{t+1}} \right)^2 + (1-\varphi)\, \mu_{t+1}^n \right], \tag{9}$$

plus the Kuhn-Tucker conditions on (3) and (7). The Euler equation for customer capital accumulation is therefore

$$\frac{\chi}{\lambda_t} \frac{AD_t}{N_t} - \mu_t^n = \mathbb{E}_t M_{t+1} \left[P_{t+1} + \frac{\chi}{2} \left(\frac{AD_{t+1}}{N_{t+1}} \right)^2 + (1-\varphi) \left(\frac{\chi}{\lambda_{t+1}} \frac{AD_{t+1}}{N_{t+1}} - \mu_{t+1}^n \right) \right]. \tag{10}$$

The Euler equation relates the marginal cost of adding one unit of search effort at time t to the marginal benefit in period $t+1$ of having λ_t additional customers, in turn consisting of higher revenues, lower adjustment costs, higher servicing costs and the discounted marginal cost of postponing to period $t+1$ the unit increase in advertising. Note that at the optimum

$$\begin{aligned} S_t &= \mathbb{E}_t \sum_{j=0}^{\infty} M_{t+j} \left[P_{t+j} N_{t+j} - \frac{\chi}{2} \frac{AD_{t+j}^2}{N_{t+j}} - \kappa N_{t+j} \right. \\ &+ \left. \mu_{t+j}^n \left((1-\varphi) N_{t+j} + \lambda_{t+j} AD_{t+j} - N_{t+j+1} \right) + \mu_{t+j}^\lambda \lambda_{t+j} AD_{t+j} \right], \end{aligned} \tag{11}$$

so that expanding S_t, I get

$$\begin{aligned} S_t &= P_t N_t - \frac{\chi}{2} \frac{AD_t^2}{N_t} - \kappa N_t + \mu_t^n \left((1-\varphi) N_t + \lambda_t AD_t - N_{t+1} \right) + \mu_t^\lambda \lambda_t AD_t \\ &+ \mathbb{E}_t M_{t+1} \left[P_{t+1} N_{t+1} - \frac{\chi}{2} \frac{AD_{t+1}^2}{N_{t+1}} - \kappa N_{t+1} \right. \qquad (12) \\ &+ \left. \mu_t^n \left((1-\varphi) N_{t+1} + \lambda_{t+1} AD_{t+1} - N_{t+2} \right) + \mu_{t+1}^\lambda \lambda_{t+1} AD_{t+1} + \ldots \right]. \qquad (13) \end{aligned}$$

Recursively substituting (10) into (12) the equilibrium, cum-dividend price of equity is

$$S_t = \left(P_t + \frac{\chi}{2} \left(\frac{AD_t}{N_t} \right)^2 + (1-\varphi)\, \mu_t^n \right) N_t, \tag{14}$$

and the ex-dividend stock price denoted by $\tilde{S}_t$ is equal to

$$\tilde{S}_t = \mu_t^n N_{t+1}. \tag{15}$$

Finally note that whenever (6) does not bind at time t (i.e., advertising is positive) the return of one unit of advertising is equal to the return on equity R_{t+1} between t and $t+1$, and is given by

$$\begin{aligned} R_{t+1} &= \frac{S_{t+1}}{\tilde{S}_t} && (16) \\ &= \frac{1}{\mu_t^n}\left(P_{t+1} + \frac{\chi}{2}\left(\frac{AD_{t+1}}{N_{t+1}}\right)^2 + (1-\varphi)\,\mu_{t+1}^n\right) && (17) \end{aligned}$$

The return on equity, as in Cochrane (1991), is the trade-off between the marginal benefit of posting an additional *ad* in period t - accrued between period t and $t+1$ - and the *ad* cost incurred in period t. Note that for a given level of past advertising, high current advertising and therefore high current advertising growth reduce future expected returns. This happens because i) through the goods market friction high advertising reduces the probability that an additional *ad* will turn into a customer (from (8), μ^n is decreasing in λ) and ii) high current advertising (and future customer capital) reduces future marginal revenues.

3.2 Household Problem

The household derives its period utility from a bundle of the numeraire good and the advertised good, and decides how much of its endowment to allocate to consumption, investment in a claim to firm profits and investment in a risk-free bond. Denote by C_0 and C_1 the representative household's consumption of the numeraire and advertised goods, respectively. Further denote by Y the endowment of the numeraire good, by ϕ the household investment in claims to the firm profits, and by θ the household investment in a risk-free asset with current price of one and gross return R^f. The claims to firm profits and the risk-free assets are in unit and zero aggregate net supply, respectively. The household's budget constraint at time $t+j$ is therefore

$$C_{0,t+j} \leq Y_{t+j} + \phi_{t+j-1}S_{t+j} + \theta_{t+j-1}R^f_{t+j} - P_{t+j}C_{1,t+j} - \phi_{t+j}\tilde{S}_{t+j} - \theta_{t+j}. \tag{18}$$

Moreover, I assume that each firm customer consumes only one unit of the advertised good, so that $C_{1,t+j} \leq N_{t+j}$. The household's intraperiod utility is given by the CES function

$$u\left(C_0; C_1\right) = \left(\left(1-\alpha\right) C_0^{\frac{\eta-1}{\eta}} + \alpha C_1^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{19}$$

where $\alpha \in [0,1]$, and $\eta \geq 0$ is the elasticity of substitution between the numeraire and manufactured goods. Finally, the household's intertemporal utility is denoted by V_t, and is specified by the recursion

$$V_t = \left\{\left(1-\beta\right) u\left(C_{0,t}; C_{1,t}\right)^{1-\frac{1}{\psi}} + \beta \mathbb{E}_t \left[V_{t+1}^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}}, \tag{20}$$

where β is the time discount factor, ψ is the elasticity of intertemporal substitution, and γ is the relative risk aversion coefficient (Kreps and Porteus (1978), Epstein and Zin (1989)). The relative price of C_1 is the marginal rate of substitution between C_0 and C_1, or

$$P = \frac{\alpha}{1-\alpha}\left(\frac{C_1}{C_0}\right)^{-\frac{1}{\eta}}. \tag{21}$$

In Appendix C I show that the stochastic discount factor is

$$M_{t+1} = \beta \left(\frac{C_{0,t+1}}{C_{0,t}}\right)^{-\frac{1}{\eta}} \left(\frac{u\left(C_{0,t+1}; C_{1,t+1}\right)}{u\left(C_{0,t}; C_{1,t}\right)}\right)^{\frac{1}{\eta}-\frac{1}{\psi}} \left[\frac{V_{t+1}}{\mathbb{E}_t\left(V_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}. \tag{22}$$

Note that, for a fixed level of past advertising, high current advertising and therefore high current advertising growth increase customer capital, decrease the numeraire good's consumption, and increase the stochastic discount factor. This and the fact that future expected returns (16) are decreasing in advertising growth implies that high advertising growth generates high negative co-variation between the stochastic discount factor and expected excess returns, and therefore high risk premia. Finally, the risk-free rate is

$$R^f_{t+1} = \frac{1}{\mathbb{E}_t\left[M_{t+1}\right]}. \tag{23}$$

3.3 Equilibrium

For each combination of the state variables $(Y; N)$, a competitive equilibrium of search in the goods market specifies policy functions for firm advertising $AD\,(Y; N)$; policy functions for household numeraire consumption $C_0\,(Y; N)$, stock holdings $\phi\,(Y; N)$ and risk-free asset holdings $\theta\,(Y; N)$; a stock price $S\,(Y; N)$, a risk-free rate $R^f\,(Y; N)$ and a relative price of the manufactured good in terms of the numeraire $P\,(Y; N)$, such that firms and households maximize their constrained objectives, markets for the numeraire and the advertised goods clear, and aggregate stock and bond markets clear. In particular, note that the equilibrium conditions in the goods market imply that

$$C_0 \quad = \quad Y - \frac{\chi}{2}\left(\frac{AD}{N}\right)^2 N, \tag{24}$$

and $C_1 = N$.

4 Results

Section 4.1 describes my calibration strategy and solution method. Section 4.2 compares the predictability results coming from simulations of the model to those coming from post-war US data. Finally, section 4.3 highlights the quantitative importance of goods market frictions in obtaining the predictability results.

4.1 Calibration and Computation

I calibrate the model at an annual frequency. In my calibration strategy I do not try to match the equity returns predictive regression coefficients found in the data, but rather show that the sign, magnitude and statistical significance of these coefficients arise naturally when the model is calibrated to match other data moments. On the firm side, Broda and Weinstein (2010) report a median annualized entry rate of new goods in consumer baskets equal to 0.25. When normalizing household search effort to unit, this entry rate in the model is equal to $\left(1 + AD^{-\vartheta}\right)^{-1/\vartheta}$, which I target to a steady-state value of 0.30 with a matching function elasticity $\vartheta = 0.57$. On the other hand, the results are not sensitive to the convex adjustment cost parameter χ, which I therefore set equal to one. On the household side, I target the home bias parameter α to 0.9 to match the one-year predictive regression coefficient of consumption growth on future consumption growth. Moreover, I use an AR(1) process in logs to describe the time-series evolution of the numeraire good's endowment, and set the persistence and volatility of the endowment process equal to 0.74 and 0.15, respectively. Finally, I set the relative risk aversion coefficient γ equal to 21. The last three parameters target a equity premium of five percent, a equity premium volatility of seventeen percent and a consumption growth volatility of 2.1 percent while retaining the main predictability results.

I borrow the remaining parameters from the literature. I choose a customer capital depreciation rate φ equal to 0.20 as in Gourio and Rudanko (2014), in the mid-range of the empirical estimates of Bronnenberg, Dubé, and Gentzkow (2012) and in the low range of the estimates of Broda and Weinstein (2010). Modeling household preferences, I set the annual discount rate β to 0.95, and the intertemporal elasticity of substitution ψ to 1.5 following Bansal and Yaron (2004). Finally, I set the elasticity of substitution parameter η equal to 0.83 following the international trade literature (Heathcote and Perri (2002), Bianchi (2009) and Huo and Ríos-Rull (2013)).

From a computational perspective, the model is challenging to solve numerically. First, the equilibrium allocations are not Pareto-optimal. A social planner confronted with the constrained equity maximization problem (5) would in fact internalize the congestion effect created by search effort, while individual firms do not. This in turn requires solving the model using its first-order conditions. Second, the non-negativity constraint on search effort renders perturbation methods not suited for this type of problems. For these reasons, I solve for the competitive search equilibrium using the globally nonlinear computational algorithm of Petrosky-Nadeau and Zhang (2013). In particular, for each point in the aggregate endowment-customer capital state space (Y_t, N_t), the algorithm solves for optimal advertising $AD_t^* = AD(Y_t, N_t)$ and the multiplier on its non-negativity constraint $\mu_t^{n*} = \mu^n(Y_t, N_t)$ from the Euler equation

$$\frac{\chi}{\lambda_t}\frac{AD_t}{N_t} - \mu^n(Y_t, N_t) = \mathbb{E}_t M_{t+1}\left[P_{t+1} + (1-\varphi)\left(\frac{\chi}{\lambda_{t+1}} - \mu^n(Y_t, N_t)\right)\right], \tag{25}$$

where both λ_t and P_t are functions of $AD(Y_t, N_t)$. Appendix D provides details on the computational algorithm.

4.2 Simulated Moments and Predictability

I simulate ten thousand samples of sixty-one annual observations, and in each simulated sample compute average advertising and consumption growth, return on equity and risk-free rate. Moreover, in each simulated sample I run predictive regressions of equity returns using consumption growth and advertising growth as predictors.

Panel A of Table 11 reports the average first moment and standard deviation of advertising and consumption growth, equity premium and risk-free rate across the simulated samples. Panel B reports the corresponding moments in post-war US data. The calibration of the model allows to reasonably match the first moments of the selected variables, and to match the volatility of consumption growth, equity premium and risk-free rate.

[Table 11 about here]

Table 12 tests the predictive power of advertising growth and past consumption growth on future consumption growth within the model, and compares the resulting coefficients to those in Table 2. As in the data, high advertising growth in the model predicts high future consumption growth. This effect in the model is however only marginally statistically significant, and quickly decays as the predictive horizon increases. On the other hand, the model calibration allows to match the predictive power of current consumption growth on future consumption growth at a predictive horizon of one year, and the decaying predictive power over longer horizons.

[Table 12 about here]

Finally, Table 13 tests the predictive power of advertising and consumption growth on future equity returns. Conditional on consumption growth, high advertising growth predicts high future returns on equity. The magnitude and statistical significance of the coefficients associated with consumption growth in the predictive regresssions is comparable to the magnitude of the coefficients in the data. The coefficients associated to advertising growth are, however, smaller than the coefficients found in the data due to the higher variance of advertising growth in the model. Overall, the model does a good job in quantitatively replicating the predictive power of consumption growth and advertising growth on future equity returns. More important, since my model calibration does not try to match the coefficient on advertising growth in returns predictive regressions, he results highlight that predictability of equity returns through advertising growth arises endogenously in this model.

[Table 13 about here]

4.3 The Quantitative Impact of Goods Market Frictions

In this section, I use the insights of the model to explore the effect of goods market frictions on predictability. In particular, I show that the congestion effect created by aggregate advertising is quantitatively important in driving both consumption and returns predictability.

As noted before, the equilibrium allocation in the decentralized economy is not Pareto-optimal. To solve for the Pareto-optimal allocation, I keep the same steady-state parametrization of the model described in section 4.1 and solve the constrained optimization problem (5) using standard value function iteration. Since firms in the centralized economy do not over-advertise to compensate the congestion effect created by aggregate advertising, the optimal amount of firm search effort in the centralized economy is as much as ten times lower than in the decentralized economy. Figure 5 shows that as a consequence the effective firm investment in customer capital (the new matches $G(AD)$) is low and almost flat.

[Figure 5 about here]

Table 14 reports estimates of the same predictive regressions coefficients of Tables (12) and (13) for the centralized economy. The results show that the non-linerity of advertising in the decentralized economy has key implications for predictability. On the consumption predictability side, firms in the centralized economy do not over-advertise to overcome the congestion effects created by aggregate advertising. Customer capital growth is flat and current advertising growth does not predict future customer capital and consumption. Similarly, on the returns predictability side a flat effective investment in customer capital reduces the non-linearity in marginal profits and marginal utility due to over-advertising, thus reducing the predictive power of advertising on future returns.

[Table 14 about here]

5 Conclusion

In this paper, I provide new evidence on the importance of advertising and goods market frictions for financial economics. I show that advertising growth predicts future consumption growth in post-war US data, and use this result to verify the core prediction of dynamic asset pricing theory that expected consumption matters for expected returns. Using advertising and consumption growth to predict excess returns on equity I show that advertising positively predicts excess returns at horizons of up to four years. Motivated by these empirical findings, I build a general equilibrium model of frictional goods markets where advertising is an investment in long-lasting customer relationships that affect the dynamics of household consumption. The calibrated model is able to replicate the predictive power of advertising growth on future consumption growth and equity returns observed in the data, and highlights the importance of frictions in the goods market to quantitatively match these predictability patterns.

The paper is part of a small literature in financial economics highlighting the importance of advertising and goods market frictions at the firm level. In this paper, I show that goods market frictions are also quantitatively relevant in the aggregate. As such, future research should be devoted to further studying the aggregate implications (i.e. the trade-off between customer capital and other forms of tangible and intangible capital) and the welfare impact of these frictions.

References

Ai, H., M. M. Croce, and K. Li (2013): "Toward a quantitative general equilibrium asset pricing model with intangible capital," *Review of Financial Studies*, 26(2), 491–530.

Bagwell, K. (2007): "The economic analysis of advertising," *Handbook of industrial organization*, 3, 1701–1844.

Bansal, R., and A. Yaron (2004): "Risks for the long run: A potential resolution of asset pricing puzzles," *The Journal of Finance*, 59(4), 1481–1509.

Belo, F., X. Lin, and S. Bazdresch (2014): "Labor Hiring, Investment, and Stock Return Predictability in the Cross Section," *Journal of Political Economy*, 122(1), 129–177.

Belo, F., X. Lin, and M. A. Vitorino (2014): "Brand capital and firm value," *Review of Economic Dynamics*, 17(1), 150–169.

Berk, J. B., R. C. Green, and V. Naik (1999): "Optimal investment, growth options, and security returns," *The Journal of Finance*, 54(5), 1553–1607.

Bianchi, J. (2009): "Overborrowing and systemic externalities in the business cycle," *Federal Reserve Bank of Atlanta Working Paper Series*, (2009-24).

Broda, C., and D. E. Weinstein (2010): "Product Creation and Destruction: Evidence and Price Implications," *The American Economic Review*, pp. 691–723.

Bronnenberg, B. J., J.-P. H. Dubé, and M. Gentzkow (2012): "The Evolution of Brand Preferences: Evidence from Consumer Migration," *American Economic Review*, 102(6), 2472–2508.

Campbell, J. Y., and J. H. Cochrane (2000): "Explaining the Poor Performance of Consumption-based Asset Pricing Models," *The Journal of Finance*, 55(6), 2863–2878.

Campbell, J. Y., and P. Perron (1991): "Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots," NBER Technical Working Papers 0100, National Bureau of Economic Research, Inc.

Campbell, J. Y., and S. B. Thompson (2008): "Predicting excess stock returns out of sample: Can anything beat the historical average?," *Review of Financial Studies*, 21(4), 1509–1531.

Clark, T. E., and K. D. West (2007): "Approximately normal tests for equal predictive accuracy in nested models," *Journal of Econometrics*, 138(1), 291–311.

Cochrane, J. H. (1991): "Production-based asset pricing and the link between stock returns and economic fluctuations," *The Journal of Finance*, 46(1), 209–237.

——— (1996): "A Cross-Sectional Test of an Investment-Based Asset Pricing Model," *Journal of Political Economy*, 104(3), 572–621.

Den Haan, W. (2013): "Inventories and the Role of Goods-Market Frictions for Business Cycles," Discussion paper, CEPR Discussion Papers.

den Haan, W. J., G. Ramey, and J. Watson (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, pp. 482–498.

Dickey, D. A., and W. A. Fuller (1979): "Distribution of the estimators for autoregressive time series with a unit root," *Journal of the American Statistical Association*, 74(366a), 427–431.

Drozd, L. A., and J. B. Nosal (2012): "Understanding international prices: Customers as capital," *The American Economic Review*, pp. 364–395.

Eisfeldt, A. L., and D. Papanikolaou (2013): "Organization Capital and the Cross-Section of Expected Returns," *The Journal of Finance*, 68(4), 1365–1406.

Epstein, L. G., and S. E. Zin (1989): "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework," *Econometrica*, pp. 937–969.

Ferson, W. E., and R. W. Schadt (1996): "Measuring fund strategy and performance in changing economic conditions," *Journal of Finance*, pp. 425–461.

Gourio, F., and L. Rudanko (2014): "Customer Capital," *The Review of Economic Studies*, 81(3), 1102–1136.

Hall, R. E. (2014): "What the cyclical response of advertising reveals about markups and other macroeconomic wedges," *Hoover Institution, Stanford University.*

Hansen, L. P., and R. J. Hodrick (1980): "Forward exchange rates as optimal predictors of future spot rates: An econometric analysis," *The Journal of Political Economy*, pp. 829–853.

Harvey, C. R. (1988): "The real term structure and consumption growth," *Journal of Financial Economics*, 22(2), 305–333.

Heathcote, J., and F. Perri (2002): "Financial autarky and international business cycles," *Journal of Monetary Economics*, 49(3), 601 – 627.

Huang, D. (2015): "Gold, Platinum, and Expected Stock Returns," Working paper.

Huo, Z., and J.-V. Ríos-Rull (2013): "Paradox of thrift recessions," Discussion paper, National Bureau of Economic Research.

Jagannathan, R., and Z. Wang (1996): "The conditional CAPM and the cross-section of expected returns," *journal of finance*, pp. 3–53.

Jermann, U. J. (1998): "Asset pricing in production economies," *Journal of Monetary Economics*, 41(2), 257–275.

Johansen, S. (1988): "Statistical analysis of cointegration vectors," *Journal of Economic Dynamics and Control*, 12(2), 231–254.

——— (1991): "Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models," *Econometrica*, pp. 1551–1580.

Kreps, D. M., and E. L. Porteus (1978): "Temporal resolution of uncertainty and dynamic choice theory," *Econometrica*, pp. 185–200.

Kuehn, L.-A., N. Petrosky-Nadeau, and L. Zhang (2012): "An equilibrium asset pricing model with labor market search," Discussion paper, National Bureau of Economic Research.

Kuehn, L.-A., M. Simutin, and J. J. Wang (2014): "A Labor Capital Asset Pricing Model," Discussion paper.

Lettau, M., and S. Ludvigson (2001a): "Consumption, aggregate wealth, and expected stock returns," *Journal of Finance*, pp. 815–849.

——— (2001b): "Resurrecting the (C) CAPM: A cross-sectional test when risk premia are time-varying," *Journal of Political Economy*, 109(6), 1238–1287.

Liu, L. X., T. M. Whited, and L. Zhang (2009): "Investment-Based Expected Stock Returns," *Journal of Political Economy*, 117(6), 1105–1139.

Nakamura, E., and J. Steinsson (2011): "Price setting in forward-looking customer markets," *Journal of Monetary Economics*, 58(3), 220–233.

Petrosky-Nadeau, N., and E. Wasmer (2015): "Macroeconomic dynamics in a model of goods, labor, and credit market frictions," *Journal of Monetary Economics*, 72, 97–113.

Petrosky-Nadeau, N., and L. Zhang (2013): "Solving the DMP model accurately," Discussion paper, National Bureau of Economic Research.

Phillips, P. C., and S. Ouliaris (1990): "Asymptotic properties of residual based tests for cointegration," *Econometrica*, pp. 165–193.

Savov, A. (2011): "Asset pricing with garbage," *The Journal of Finance*, 66(1), 177–201.

Storesletten, K., J.-V. R. Rull, and Y. Bai (2011): "Demand Shocks that Look Like Productivity Shocks," in *2011 Meeting Papers*, no. 99. Society for Economic Dynamics.

Vitorino, M. A. (2014): "Understanding the Effect of Advertising on Stock Returns and Firm Value: Theory and Evidence from a Structural Model," *Management Science*, 60(1), 227–245.

A Cointegration Tests

In Table 9, I report two sets of tests for cointegration between real, per-capita advertising expenditures and consumption. In Panel A, I use the Phillips and Ouliaris (1990) procedure to test for a unit root in the residual of a regression of advertising expenditures on consumption, assuming no trend in the residuals. Panel A of Table 9 reports the Dickey and Fuller (1979) t-statistic for a unit root in the residuals using lags from one to four years, and the associated five and ten percent critical values. The null hypothesis of no cointegrating relationship can never be rejected at any horizon. I use the procedure in Campbell and Perron (1991) to determine the appropriate number of lags of first differences in the regression of residuals on lagged residuals and lagged first differences of residuals, and the results of this procedure suggest that the optimal number of lag is three years. The results of Panel A provide evidence against cointegration at the optimal lag length.

As a second test, I apply the Johansen (1988, 1991) procedure to estimate the number of cointegrating relationships between advertising expenditures and consumption, assuming that the cointegrating relation should be characterized by an unrestricted constant.[7] The Johansen trace statistic tests the null hypothesis $H_0 = r$ of at most r cointegrating relations in the data against the alternative hypothesis of p cointegrating

[7]This assumption is common in modeling macroeconomic variables. See Johansen (1988, 1991) for details.

relations, where p is the number of variables (two in this case), and the null hypothesis is rejected at the five percent confidence level if the trace statistics is larger than its respective critical value. Table 9, Panel B, shows that the test can never reject the null hypothesis of zero cointegrating relationships between advertising and consumption at any of the lags considered.

[**Table 9 about here**]

Despite the weak evidence about cointegration between advertising expenditures and consumption, in Table 10 I re-estimate the consumption growth predictive regressions of Table 2, Panel A, using a vector-error-correction model (VECM). The estimated VECM corrects the predictive regressions with a cointegrating residual capturing deviations of either consumption or advertising from their long-run common trend. The results show that correcting for this cointegrating residual decreases the predictive power of advertising at a two-year horizon, but leaves leaves the predictive power of advertising unchanged at a one-year horizon.

[**Table 10 about here**]

B Advertising Expenditures Are Not Long-Run Risk

In this Section, I claim that the time series properties of aggregate advertising growth make this variable a quantitatively different source of consumption dynamics than the aggregate consumption growth risk in the Bansal and Yaron (2004) long-run risk model. The Bansal and Yaron (2004) long-run risk model specifies the following process for consumption growth (for consistency with their model I use Δc_{t+1} to denote the consumption growth rate $\Delta c_{t \to t+1}$):

$$\Delta c_{t+1} = \kappa + x_t + \sigma \eta_{t+1}, \tag{26}$$

$$x_{t+1} = \rho x_t + \phi_e \sigma e_{t+1}, \tag{27}$$

$$e_{t+1}, \eta_{t+1} \sim N.i.i.d\,(0,1), \tag{28}$$

where the shocks e_{t+1} and η_{t+1} are mutually independent. In their model, x_t is a small and persistent predictable component that determines the expected growth rate of consumption and ρ is the persistence of this predictable component, calibrated to a monthly $\rho = 0.979$ (annual $\rho_{ann} = 0.775$) to replicate the annualized volatility and autocorrelation of aggregate consumption growth.

On the other hand, the VAR specification from Panel A of Table 5 implies the following relation between consumption growth, advertising expenditures and their lagged values:

$$\Delta c_{t+1} = \alpha_c + \gamma_c \Delta a_t + u_{c,t+1}, \tag{29}$$

$$\Delta a_{t+1} = \alpha_a + \beta_a \Delta c_t + \gamma_a \Delta a_t + u_{a,t+1}, \tag{30}$$

$$u_{c,t+1}, u_{a,t+1}, \sim N(0, \Sigma), \tag{31}$$

with Σ the variance-covariance matrix of the residuals. For simplicity, I omit the VAR coefficients that are not statistically significant. The point estimate of the coefficient γ_c in Equation (29) is 0.127, and its standard deviation is 0.050. The point estimates for the coefficients β_a and γ_a in Equation (30) are 0.679 and -1.147, respectively, and their standard deviations are 0.144 and 0.456, respectively.

The following analysis is to test whether, given these estimates, Equations (29) and (30) can respectively be re-written as Equations (26) and (27), that is whether advertising growth captures the long-run persistent component of consumption growth that generates long-run risk. First, define $\Delta\tilde{a}_t \equiv \gamma_c \Delta a_t$, so that (29)-(30) can be re-written as

$$\Delta c_{t+1} = \alpha_c + \Delta\tilde{a}_t + u_{c,t+1}, \tag{32}$$

$$\Delta\tilde{a}_{t+1} = \gamma_c \alpha_a + \gamma_c \beta_a \Delta c_t + \gamma_c \Delta\tilde{a}_t + \gamma_c u_{a,t+1} \tag{33}$$

Testing if (30) is equivalent to the long-run risk equation (27) then means simultaneously testing for $\gamma_c \alpha_a = \gamma_c \beta_a = 0$ and $\gamma_c = \rho_{ann} = 0.775$. Since the point estimate for γ_c in Table 5 is however equal to 0.127 with a 95 percent confidence interval of $[0.029; 0.225]$, I cannot reject the null hypothesis that $\gamma_c \neq \rho_{ann}$. This suggests that advertising growth predicts a component of aggregate consumption growth not captured by long-run risk.

C Derivation of the Stochastic Discount Factor

The derivative of (20) with respect to $C_{0,t}$ is

$$\frac{\partial V_t}{\partial C_{0,t}} = (1-\beta)(1-\alpha) \quad V_t^{\frac{1}{\psi}} u(C_{0,t};C_{1,t})^{\frac{1}{\eta}-\frac{1}{\psi}} C_{0,t}^{-\frac{1}{\eta}}. \tag{34}$$

The derivative of (20) with respect to $C_{0,t+1}$ is

$$\frac{\partial V_t}{\partial C_{0,t+1}} = V_t^{\frac{1}{\psi}} \beta \mathbb{E}_t \left[V_{t+1}^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}-1} V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial C_{0,t+1}}. \tag{35}$$

Replacing $\partial V_{t+1}/\partial C_{0,t+1}$ by (34) evaluated at $t+1$, I get

$$\begin{aligned} M_{t+1} &= \frac{\partial V_t/\partial C_{0,t+1}}{\partial V_t/\partial C_{0,t}} && (36)\\ &= \beta\left(\frac{C_{0,t+1}}{C_{0,t}}\right)^{-\frac{1}{\eta}}\left(\frac{u(C_{0,t+1};C_{1,t+1})}{u(C_{0,t};C_{1,t})}\right)^{\frac{1}{\eta}-\frac{1}{\psi}}\left[\frac{V_{t+1}}{\mathbb{E}_t\left(V_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}. && (37)\end{aligned}$$

D Computational Algorithm

The state space consists of aggregate endowment and customer capital, (Y_t, N_t), and the objective is to solve for optimal advertising $AD_t^* = AD(Y_t,N_t)$ and the multiplier on its non-negativity constraint $\mu_t^{n*} = \mu^n(Y_t,N_t)$ from the functional Euler equation

$$\frac{\chi}{\lambda_t}\frac{AD_t}{N_t} - \mu^n(Y_t,N_t) = \mathbb{E}_t M_{t+1}\left[P_{t+1} + (1-\varphi)\left(\frac{\chi}{\lambda_{t+1}} - \mu^n(Y_t,N_t)\right)\right], \tag{38}$$

where both λ_t and P_t are functions of $AD(Y_t,N_t)$. The algorithm works as follows. I start by approximating the left-hand side of (38) with a function

$$\mathcal{E}_t \equiv \mathcal{E}(Y_t,N_t) = \mathbb{E}_t M_{t+1}\left[P_{t+1} + (1-\varphi)\left(\frac{\chi}{\lambda_{t+1}} - \mu^n(Y_t,N_t)\right)\right]. \tag{39}$$

Since the function $\mathcal{E}_t$ is defined over the grid (Y_t, N_t), I can similarly define

$$\tilde{\mathcal{E}}(Y_t, N_t) \equiv \frac{1}{\chi}\left(N_t \mathcal{E}(Y_t, N_t)\right). \tag{40}$$

Finally, since $\lambda = \left(1 + AD^{\vartheta}\right)^{-1/\vartheta}$, I calculate a guess $\tilde{AD}(Y_t, N_t)$ for the policy function $AD(Y_t, N_t)$ by solving

$$\tilde{AD}(Y_t, N_t)\left(1 + \tilde{AD}(Y_t, N_t)^{\vartheta}\right)^{\frac{1}{\vartheta}} = \tilde{\mathcal{E}}(Y_t, N_t), \tag{41}$$

so that solving function for effort is:[8]

$$\tilde{AD}(Y_t, N_t) = 2^{-\frac{1}{\vartheta}}\left(\sqrt{4\tilde{\mathcal{E}}(Y_t, N_t)^{\vartheta} + 1} - 1\right)^{\frac{1}{\vartheta}}. \tag{42}$$

If $AD(Y_t, N_t) > 0$, then the non-negativity constraint on effort is not binding, $AD(Y_t, N_t) = \tilde{AD}(Y_t, N_t)$ and $\mu^n(Y_t, N_t) = 0$. If instead $\tilde{AD}(Y_t, N_t) \leq 0$, then $AD(Y_t, N_t) = 0$ and $\mu^n(Y_t, N_t) = -\tilde{\mathcal{E}}(Y_t, N_t)$.

[8]The smaller root of Equation (41),

$$\tilde{E}(Y_t, N_t) = 2^{-\frac{1}{\vartheta}}\left(-\sqrt{4\tilde{\mathcal{E}}(Y_t, N_t)^{\vartheta} + 1} - 1\right)^{\frac{1}{\vartheta}},$$

is always negative.

E Figures and Tables

Figure 1: Expenditures in Physical and Non-Physical Advertising in the U.S., 1950-2010
Physical advertising is the sum of advertising on newspapers, periodicals, magazines, direct mail, yellow pages, farm publications, billboards and business papers. Non-physical advertising includes radio, television and internet. Total advertising is the sum of physical and non-physical advertising. The advertising data for the years 1900-2007 are hand-collected by Robert J. Coen from the advertising company Erickson-McCann and can be found on Douglas Galbi's website. For the years 2007 to 2010, Hall (2014) updates this dataset using revenue data from companies in the information sector published by the Census Bureau. These data are no longer available, but can be found on Hall's website. All the data are expressed in 2005 US billion dollars using the producer price index.

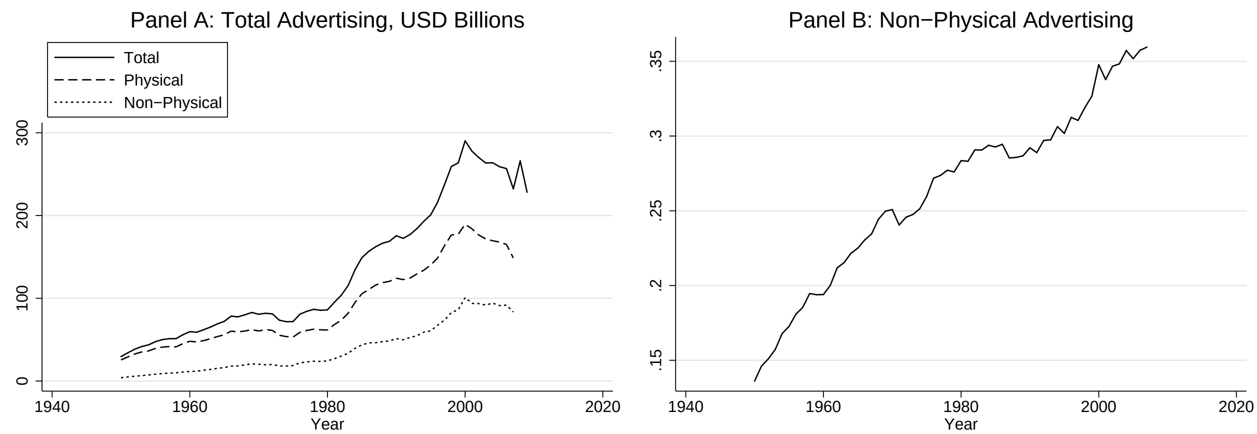

Figure 2: Per-Capita Consumption and Advertising in the U.S., 1950-2010

Consumption is Personal Consumption Expenditures from NIPA Tables, expressed in 2005 US dollars using the Consumer Price Index. Advertising is the sum of advertising on newspapers, periodicals, magazines, direct mail, yellow pages, farm publications, billboards, business papers, radio, television and internet. The advertising data for the years 1900-2007 are hand-collected by Robert J. Coen from the advertising company Erickson-McCann and can be found on Douglas Galbi's website. For the years 2007 to 2010, Hall (2014) updates this dataset using revenue data from companies in the information sector published by the Census Bureau. These data are no longer available, but can be found on Hall's website. Advertising expenditures is expressed in 2005 US dollars using the producer price index.

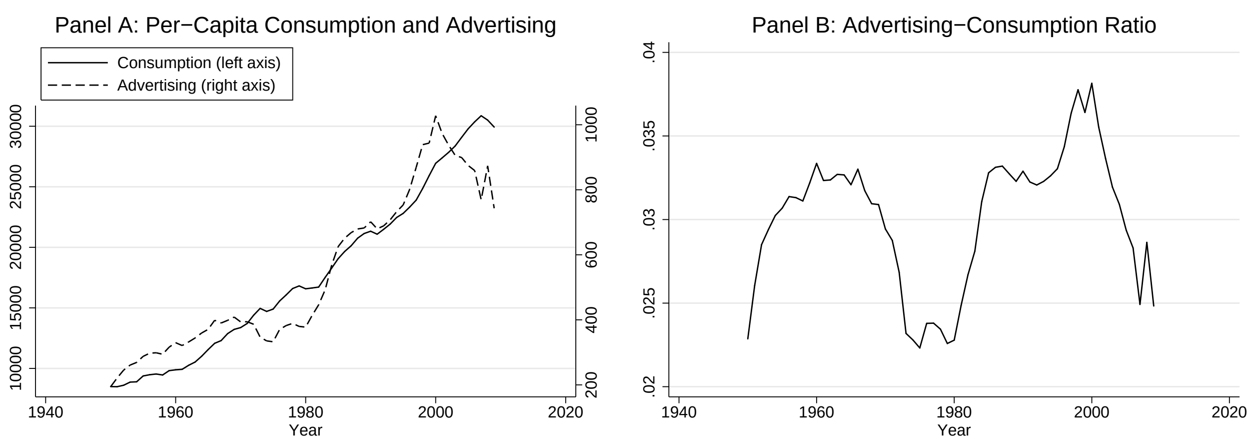

Figure 3: Advertising Expenditures Growth, Consumption Growth and Excess Returns in the U.S., 1950-2010

Consumption is Personal Consumption Expenditures from NIPA Tables. The advertising data for the years 1900-2007 are hand-collected by Robert J. Coen from the advertising company Erickson-McCann and can be found on Douglas Galbi's website. For the years 2007 to 2010, Hall (2014) updates this dataset using revenue data from companies in the information sector published by the Census Bureau. These data are no longer available, but can be found on Hall's website. Advertising is the sum of advertising on newspapers, periodicals, magazines, direct mail, yellow pages, farm publications, billboards, business papers, radio, television and internet. Excess returns are yearly returns on the S&P 500 minus the one-year interest rate from Robert Shiller's website.

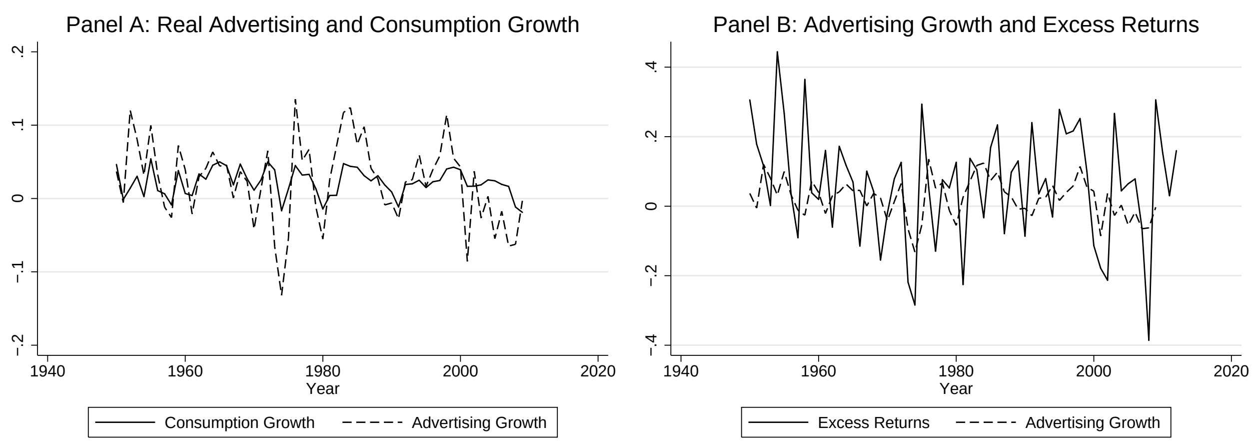

Figure 4: Coefficient Estimates in Out-of-Sample Excess Returns Predictive Regressions, 1980-2010
Growth in advertising expenditures and consumption are used to predict excess returns in the next year, where excess returns are the yearly returns on the S&P 500 minus the one-year interest rate from Robert Shiller's website. Panel A reports point estimates and 95% confidence intervals for the slope coefficient of advertising expenditures, using a regression with a rolling window of 50 periods (years). Panel B reports point estimates and 95% confidence intervals for the slope coefficient of relative advertising expenditures, using an expanding regression with an initial length of 50 periods (years).

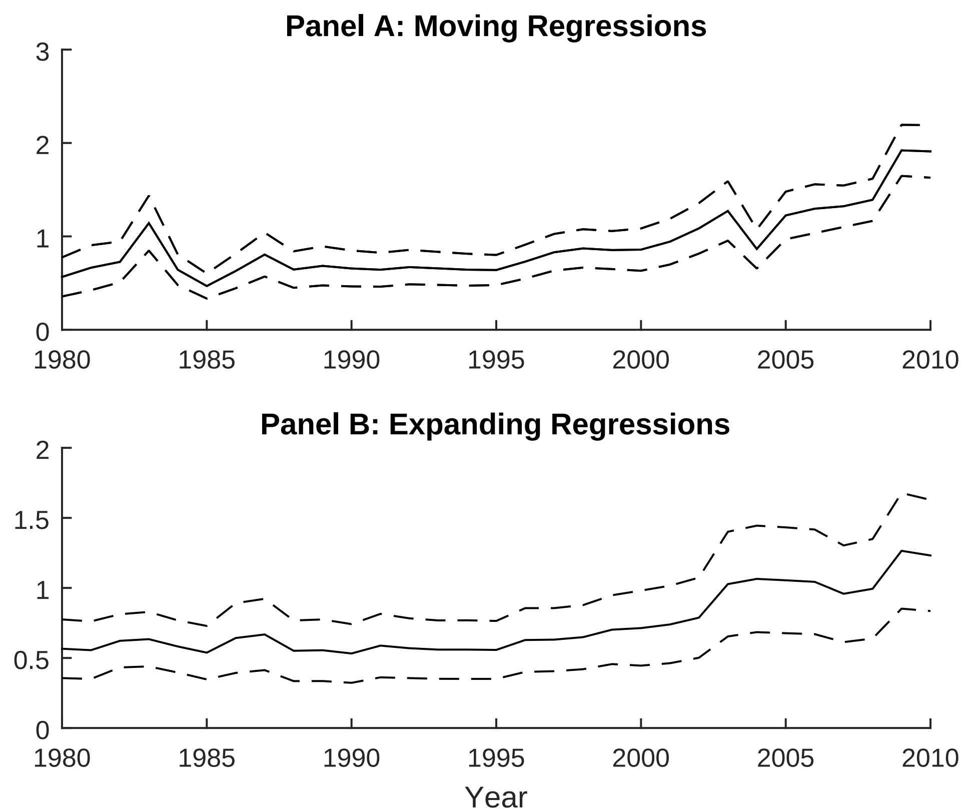

Figure 5: Customer Capital Investment

The Figure compares customer capital investment $G(AD)$ in the decentralized economy with investment in the decentralized economy. Panel A shows the investment as a function of customer capital N, for the lowest possible realization of the endowment process Y. Panel B shows the investment as a function of customer capital N, for the highest possible realization of the endowment process Y.

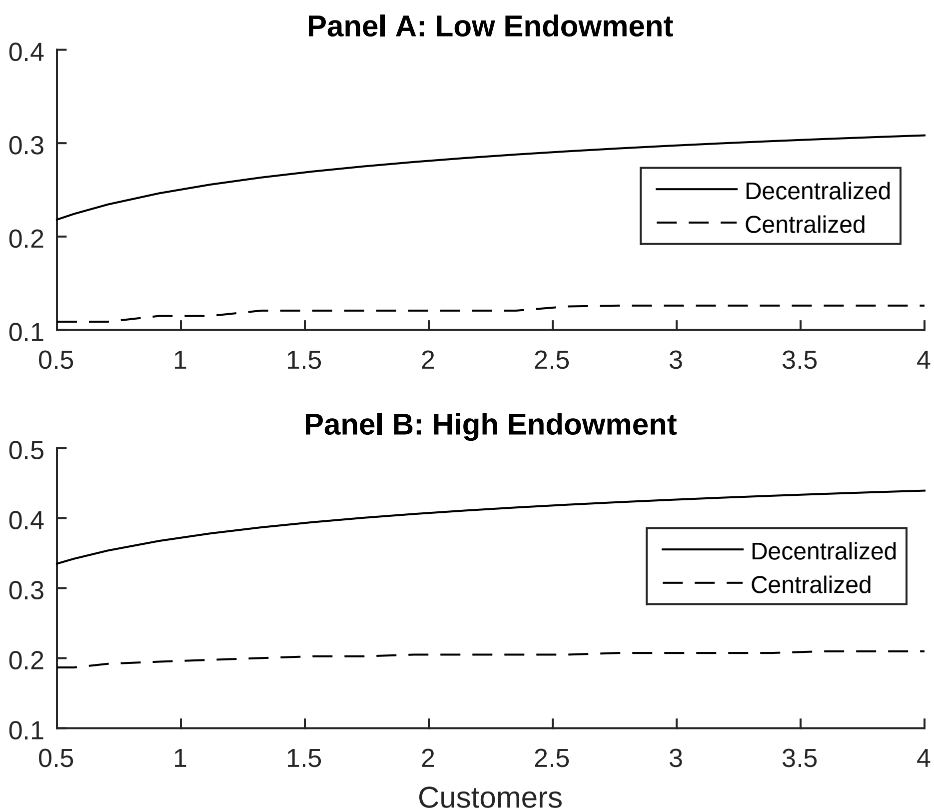

Table 1: Summary Statistics for Predictors, Post-War Period

The Table gives summary statistics for advertising expenditures growth (Δa_t), consumption growth (Δc_t), as well as other known stock returns predictors. Consumption is Personal Consumption Expenditures from NIPA Tables. Advertising is the sum of advertising on newspapers, periodicals, magazines, direct mail, yellow pages, farm publications, billboards, business papers, radio, television and internet. The advertising data for the years 1900-2007 are hand-collected by Robert J. Coen from the advertising company Erickson-McCann and can be found on Douglas Galbi's website. For the years 2007 to 2010, Hall (2014) updates this dataset using revenue data from companies in the information sector published by the Census Bureau. These data are no longer available, but can be found on Hall's website. $\log dp_t$ and $\log pe_t$ are respectively the log price-dividend ratio and the cyclically-adjusted log price-earnings ratio, both from Robert Shiller's website. pay_t is the net payout yield from Michael Roberts's website. The default spread def_t is the difference between the yield of Baa and Aaa corporate bonds, while the term spread $term_t$ is the difference between the yield of a 10 year constant maturity U.S. government bond and the yield on a 3 month constant maturity U.S. T-bill. The inflation rate π_t is the growth rate of the Consumer Price Index. The data for def_t , $term_t$ and π_t comes from FRED. The data for the consumption-wealth cointegrating residual cay_t comes from Martin Lettau's website. ADF is the augmented Dickey and Fuller (1979) test statistic.

Variable	Mean	St. Dev.	Max.	Min.	Corr. Δa_t	AR(1)	t-stat	ADF	p-value	Range
Δa_t	0.023	0.056	0.136	-0.132	1.000	0.387	3.035	-4.361	0.000	1950-2010
Δc_t	0.022	0.018	0.054	-0.019	-0.060	0.269	2.044	-4.446	0.000	1950-2010
$\log dp_t$	-3.501	0.423	-2.669	-4.448	0.122	0.932	19.016	-1.843	0.359	1950-2012
$\log pe_t$	2.755	0.410	3.833	1.985	-0.079	0.843	11.858	-2.651	0.083	1950-2012
pay_t	0.115	0.021	0.161	0.054	0.149	0.787	12.451	-3.026	0.033	1950-2010
def_t	0.943	0.420	2.320	0.000	-0.105	0.838	18.452	-3.586	0.006	1950-2014
$term_t$	1.810	1.072	3.490	-0.060	0.122	0.480	2.496	-4.034	0.001	1950-2014
π_t	0.037	0.029	0.139	-0.007	-0.315	0.739	8.655	-3.079	0.028	1950-2013
cay_t	-0.000	0.017	0.033	-0.036	0.272	0.887	14.948	-1.360	0.601	1952-2013

Table 2: Consumption Growth and Excess Returns Predictability, Post-War Period

The Table shows coefficient estimates for cumulative consumption growth ($\Delta c_{t\to t+\tau}$) and excess returns ($r^x_{t\to t+\tau}$) predictive regressions using lagged advertising expenditures growth ($\Delta a_{t-1\to t}$) and consumption growth ($\Delta c_{t-1\to t}$) as predictors. Excess returns are yearly returns on the S&P 500 minus the one-year interest rate from Robert Shiller's website. The t-statistics in parentheses are computed using Hansen and Hodrick (1980) standard errors. R^2_{adj} and F are the adjusted R-squared and F-statistics, respectively.

		Panel A: Consumption Growth				Panel B: Excess Returns			
		$\Delta c_{t\to t+1}$	$\Delta c_{t\to t+2}$	$\Delta c_{t\to t+3}$	$\Delta c_{t\to t+4}$	$r^x_{t\to t+1}$	$r^x_{t\to t+2}$	$r^x_{t\to t+3}$	$r^x_{t\to t+4}$
(1)	$\Delta a_{t-1\to t}$	0.129	0.160	0.128	0.085	0.293	0.476	0.956	0.910
		(3.25)	(2.10)	(1.20)	(0.67)	(0.78)	(0.76)	(1.12)	(0.83)
	R^2_{adj}	0.143	0.074	0.018	-0.007	-0.007	-0.005	0.015	0.001
(2)	$\Delta c_{t-1\to t}$	0.266	0.183	0.043	-0.069	-2.127	-3.291	-4.020	-6.191
		(2.03)	(0.75)	(0.14)	(-0.19)	(-1.92)	(-1.84)	(-1.74)	(-2.13)
	R^2_{adj}	0.051	-0.006	-0.018	-0.018	0.040	0.046	0.045	0.067
(3)	$\Delta a_{t-1\to t}$	0.127	0.201	0.195	0.161	1.203	1.900	2.936	3.626
		(2.47)	(2.00)	(1.33)	(0.91)	(2.92)	(2.88)	(3.18)	(2.93)
	$\Delta c_{t-1\to t}$	0.010	-0.214	-0.337	-0.380	-4.404	-6.887	-9.579	-13.477
		(0.07)	(-0.71)	(-0.79)	(-0.74)	(-3.42)	(-3.46)	(-3.62)	(-3.72)
	R^2_{adj}	0.128	0.067	0.016	-0.010	0.128	0.148	0.215	0.224
	F	5.049	2.186	0.859	0.397	6.037	6.219	6.829	6.718

Table 3: Excess Returns Predictive Regressions, Post-War Period

The Table shows coefficient estimates for cumulative excess returns ($r^x_{t\to t+\tau}$) predictive regressions using lagged growth in advertising ($\Delta a_{t-1\to t}$, conditional on lagged consumption growth $\Delta c_{t-1\to t}$) as well as other variables, as predictors. Excess returns are yearly returns on the S&P 500 minus the one-year interest rate from Robert Shiller's website. The reported t-statistics are computed using Hansen and Hodrick (1980) standard errors. R^2_{adj} is the adjusted R-squared statistics.

	$r^x_{t\to t+1}$			$r^x_{t\to t+2}$			$r^x_{t\to t+3}$			$r^x_{t\to t+4}$		
	Coeff.	t-stat	R^2_{adj}	Coeff.	t-stat	R^2_{adj}	Coeff.	t-stat	R^2_{adj}	Coeff.	t-stat	R^2_{adj}
$\Delta a_{t-1\to t}$	1.20	2.92	0.13	1.90	2.88	0.15	2.94	3.18	0.21	3.63	2.93	0.22
$\Delta c_{t-1\to t}$	-4.40	-3.42		-6.89	-3.46		-9.58	-3.62		-13.48	-3.72	
$\log dp_{t-1}$	0.10	2.01	0.05	0.18	2.04	0.09	0.25	1.71	0.11	0.38	1.81	0.16
$\log pe_{t-1}$	-0.04	-0.86	-0.01	-0.07	-0.82	-0.00	-0.11	-0.82	0.01	-0.23	-1.15	0.04
pay_{t-1}	2.51	2.74	0.09	3.85	2.27	0.10	4.96	1.80	0.10	6.64	1.76	0.12
def_{t-1}	0.01	0.14	-0.02	-0.04	-0.45	-0.01	-0.04	-0.31	-0.01	-0.02	-0.11	-0.02
$term_{t-1}$	0.07	2.65	0.16	0.11	2.36	0.19	0.14	2.10	0.19	0.19	2.08	0.20
π_{t-1}	-0.44	-0.65	-0.01	-1.12	-0.91	0.00	-1.31	-0.69	-0.00	-0.50	-0.19	-0.02
cay_{t-1}	2.55	2.30	0.05	5.11	2.65	0.11	8.11	2.96	0.19	11.33	3.23	0.25

Table 4: Tri-Variate Excess Returns Predictive Regressions, Post-War Period

The Table shows coefficient estimates for cumulative excess returns ($r^x_{t\rightarrow t+\tau}$) predictive regressions using the lagged growth in advertising ($\Delta a_{t-1\rightarrow t}$) and consumption ($\Delta c_{t-1\rightarrow t}$, omitted in the Table) in tri-variate regressions with other predictors. Excess returns are yearly returns on the S&P 500 minus the one-year interest rate from Robert Shiller's website. The reported t-statistics are computed using Hansen and Hodrick (1980) standard errors. R^2_{adj} and F are the adjusted R-squared and F-statistics, respectively.

	$r^x_{t\rightarrow t+1}$						$r^x_{t\rightarrow t+2}$					
	Coeff. Δa_{t-1}	t-stat Δa_{t-1}	Coeff.	t-stat	R^2_{adj}	F	Coeff. Δa_{t-1}	t-stat Δa_{t-1}	Coeff.	t-stat	R^2_{adj}	F
$\log dp_{t-1}$	1.05	2.40	0.07	1.54	0.14	4.89	1.57	2.26	0.14	1.85	0.19	4.79
$\log pe_{t-1}$	1.20	2.78	-0.00	-0.08	0.11	4.01	1.87	2.74	-0.02	-0.24	0.13	4.12
$\log pay_{t-1}$	1.01	2.39	1.68	2.02	0.16	5.65	1.60	2.38	2.59	1.79	0.18	5.06
def_{t-1}	1.28	3.16	-0.05	-1.23	0.13	4.54	2.11	3.47	-0.14	-2.15	0.18	6.69
$term_{t-1}$	1.48	2.47	0.05	1.94	0.27	4.57	1.49	1.53	0.08	1.88	0.22	2.76
π_{t-1}	1.21	3.07	-0.93	-1.68	0.16	5.67	1.87	3.06	-2.00	-2.13	0.21	6.88
cay_{t-1}	1.01	2.26	1.83	1.74	0.16	6.11	1.45	2.15	4.06	2.36	0.23	7.29

	$r^x_{t\rightarrow t+3}$						$r^x_{t\rightarrow t+4}$					
	Coeff. Δa_{t-1}	t-stat Δa_{t-1}	Coeff.	t-stat	R^2_{adj}	F	Coeff. Δa_{t-1}	t-stat Δa_{t-1}	Coeff.	t-stat	R^2_{adj}	F
$\log dp_{t-1}$	2.54	2.74	0.17	1.47	0.25	4.71	2.91	2.32	0.26	1.57	0.29	4.60
$\log pe_{t-1}$	2.88	3.13	-0.03	-0.29	0.20	4.48	3.43	2.81	-0.08	-0.52	0.22	4.41
$\log pay_{t-1}$	2.61	2.83	2.87	1.31	0.24	4.85	3.10	2.52	4.34	1.44	0.26	4.85
def_{t-1}	3.20	3.68	-0.17	-1.78	0.25	6.37	3.82	3.24	-0.17	-1.16	0.24	5.36
$term_{t-1}$	2.18	1.63	0.10	1.57	0.25	2.50	2.83	1.81	0.15	1.70	0.32	3.43
π_{t-1}	2.89	3.34	-2.26	-1.65	0.26	6.47	3.85	3.63	-2.82	-1.61	0.33	8.33
cay_{t-1}	2.17	2.50	5.92	2.63	0.31	7.71	2.58	2.60	7.91	2.97	0.38	10.04

Table 5: VAR Model for Advertising and Consumption Growth, Post-War Period

The Table shows coefficient estimates for a Vector-Autoregressive (VAR) model of advertising expenses and consumption growth (Δa and Δc, respectively). The t-statistics are in parentheses. In each equation, R^2 and F are the R-squared and F-statistics, respectively.

	Panel A: One Lag		Panel B: Two Lags	
	$\Delta a_{t\to t+1}$	$\Delta c_{t\to t+1}$	$\Delta a_{t\to t+1}$	$\Delta c_{t\to t+1}$
$\Delta a_{t-1\to t}$	0.679	0.127	0.515	0.124
	(4.71)	(2.52)	(3.23)	(2.16)
$\Delta a_{t-2\to t-1}$			0.242	-0.0167
			(1.49)	(-0.28)
$\Delta c_{t-1\to t}$	-1.417	0.0105	-1.119	0.0681
	(-3.11)	(0.07)	(-2.39)	(0.41)
$\Delta c_{t-2\to t-1}$			-1.180	0.00771
			(-2.38)	(0.04)
R^2	0.274	0.158	0.336	0.170
F	11.12	5.521	7.333	2.973

Table 6: VAR Model for Advertising and Consumption Growth, 1922-2009 and 1982-2009
The Table shows coefficient estimates for a Vector-Autoregressive (VAR) model of advertising expenses and consumption growth (Δa and Δc, respectively) across different samples. The t-statistics are in parentheses. In each equation, R^2 and F are the R-squared and F-statistics, respectively.

	Panel A: 1922-2009				Panel B: 1982-2009			
	One Lag		Two Lags		One Lag		Two Lags	
	$\Delta a_{t\to t+1}$	$\Delta c_{t\to t+1}$	$\Delta a_{t\to t+1}$	$\Delta c_{t\to t+1}$	$\Delta a_{t\to t+1}$	$\Delta c_{t\to t+1}$	$\Delta a_{t\to t+1}$	$\Delta c_{t\to t+1}$
$\Delta a_{t-1\to t}$	0.352	0.0666	0.301	0.0433	0.783	0.166	0.454	0.117
	(2.97)	(1.16)	(2.39)	(0.70)	(4.10)	(3.37)	(2.73)	(2.25)
$\Delta a_{t-2\to t-1}$			0.0602	0.0572			0.856	0.137
			(0.49)	(0.95)			(4.53)	(2.31)
$\Delta c_{t-1\to t}$	-0.242	0.140	-0.190	0.159	-1.312	0.224	-1.614	0.155
	(-0.95)	(1.13)	(-0.73)	(1.25)	(-1.95)	(1.29)	(-2.36)	(0.72)
$\Delta c_{t-2\to t-1}$			-0.124	-0.0885			-1.900	-0.261
			(-0.47)	(-0.70)			(-3.15)	(-1.38)
R^2	0.0966	0.0612	0.0825	0.0676	0.383	0.543	0.651	0.604
F	4.759	2.900	1.978	1.596	8.992	17.24	13.06	10.70

Table 7: Consumption Growth Predictive Regressions, Post-War Period

The Table shows coefficient estimates for cumulative consumption growth ($\Delta c_{t\to t+\tau}$) predictive regressions using lagged consumption growth ($\Delta c_{t-1\to t}$), advertising expenditures growth ($\Delta a_{t-1\to t}$), as well as the lagged short-term interest rate (r^{3m}_{t-1}) and lagged term spread from $t-1$ to $t-1+\tau$ (r^{τ}_{t-1}) (Harvey (1988)) as predictors. The short-term interest rate is the yield on a 3 month constant maturity U.S. T-bill, and the term spread is the difference between a U.S. government bond with constant maturity τ and the short-term interest rate. The data is from FRED. The t-statistics in parentheses are computed using Hansen and Hodrick (1980) standard errors. R^2_{adj} and F are the adjusted R-squared and F-statistics, respectively.

		$\Delta c_{t\to t+1}$	$\Delta c_{t\to t+2}$	$\Delta c_{t\to t+3}$	$\Delta c_{t\to t+4}$
(1)	r^{3m}_{t-1}	0.001	0.001	0.003	0.071
		(0.85)	(0.26)	(0.81)	(0.70)
	r^{τ}_{t-1}	0.020	0.030	0.023	0.063
		(1.82)	(2.81)	(1.80)	(0.65)
	R^2_{adj}	0.172	0.272	0.146	0.072
	F	2.590	3.669	1.588	0.654
(2)	$\Delta a_{t-1\to t}$	0.149	0.395	0.541	0.629
		(2.36)	(3.51)	(3.07)	(3.25)
	$\Delta c_{t-1\to t}$	0.241	-0.281	-0.786	-1.104
		(1.21)	(-0.76)	(-1.38)	(-1.65)
	r^{3m}_{t-1}	-0.000	-0.002	-0.001	0.023
		(-0.38)	(-1.17)	(-0.43)	(0.29)
	r^{τ}_{t-1}	0.010	0.013	0.009	0.022
		(1.14)	(1.59)	(0.87)	(0.29)
	R^2_{adj}	0.475	0.519	0.351	0.305
	F	7.511	6.196	3.154	2.599

Table 8: Out-of-Sample Excess Returns Predictive Regressions

The Table shows the out-of-sample performance of cumulative excess returns ($r^x_{t\to t+\tau}$) predictive regressions using lagged growth in advertising ($\Delta a_{t-1\to t}$) and consumption ($\Delta c_{t-1\to t}$) as predictors. The statistic R^2_{ws} is the within-sample adjusted R-squared statistic. Out-of-sample moving regressions use a 30-year rolling window to predict cumulative excess returns at different horizons, starting from 1980. Out-of-sample expanding regressions use the initial 1950-1980 sample to predict cumulative excess returns at different horizons. The procedure is then repeated by expanding the sample in one-year steps until the full 1950-2010 sample is obtained. The statistic $R^2_{os}$is the out-of-sample R-squaredd statistic detailed in Campbell and Thompson (2008). The Newey-West (NW) t-statistics are obtained from regressing the adjusted-MSPE statistics of Clark and West (2007) on a constant. The test is a one-side test for a zero coefficient.

	Within-Sample	Out-of-Sample Moving		Out-of-Sample Expanding	
Horizon (years)	R^2_{ws}	R^2_{os}	NW t-stat.	R^2_{os}	NW t-stat.
1	0.128	0.071	1.216	0.066	1.385
2	0.148	0.129	1.916	0.114	2.017
3	0.215	0.153	2.005	0.147	2.419
4	0.224	0.139	2.349	0.177	3.178

Table 9: Philips-Ouliaris and Johansen Tests for Cointegration

In Panel A, the Dickey and Fuller (1979) test statistics is applied to the fitted residuals of a regression of per-capita real advertising expenditures on per-capita real consumption. No trend is assumed in the residuals. The procedure in Campbell and Perron (1991) is used to to determine the number of lags of first differences in the regression of residuals on lagged residuals and lagged first differences of residuals. In Panel B, I apply the Johansen (1988, 1991) trace statistic assuming that the relation between consumption and advertising expenditures in the data is governed by VAR model with unrestrticted constant. The null hypothesis $H_0 = r$ of at most r cointegrating relationships in the data is rejected at the 5% confidence level if the trace statistics is larger than the respective critical value.

Panel A: Philips-Ouliaris Test					
Dickey-Fuller t-statistic				Critical Values	
Lag=1	Lag=2	Lag=3	Lag=4	5%	10%
-1.346	-2.190	-2.290	-2.389	-2.926	-2.598

Panel B: Johansen Trace Statistic					
Johansen Trace Statistic				Critical Value	$H_0 = r$
Lag=1	Lag=2	Lag=3	Lag=4	5%	$r =$
12.591	6.594	4.321	5.978	15.41	0
4.086	2.426	0.263	0.472	3.76	1

Table 10: Vector-Error-Correction Model for Consumption Growth Predictions, Post-War Period

The Table shows coefficient estimates for cumulative consumption growth ($\Delta c_{t \to t+\tau}$) predictive regressions using a Vector-Error-Correction model including lagged advertising growth ($\Delta a_{t-1 \to t}$), consumption growth ($\Delta c_{t-1 \to t}$), and their long-run cointegrating residual $\varepsilon(a,c)_{t-1}$ as predictors. The t-statistics in parentheses are computed using Hansen and Hodrick (1980) standard errors. R^2_{adj} and F are the adjusted R-squared and F-statistics, respectively.

	$\Delta c_{t \to t+1}$	$\Delta c_{t \to t+2}$	$\Delta c_{t \to t+3}$	$\Delta c_{t \to t+4}$
$\Delta a_{t-1 \to t}$	0.112	0.162	0.151	0.111
	(2.06)	(1.56)	(1.01)	(0.62)
$\Delta c_{t-1 \to t}$	0.027	-0.160	-0.274	-0.309
	(0.17)	(-0.53)	(-0.65)	(-0.60)
$\varepsilon(a,c)_{t-1}$	-0.012	-0.032	-0.044	-0.054
	(-0.86)	(-0.98)	(-0.87)	(-0.81)
R^2_{adj}	0.124	0.080	0.031	0.007
F	3.545	1.669	0.756	0.446

Table 11: Model Simulated Moments

	Panel A: Model		Panel B: Data	
	Mean	St. Dev	Mean	St. Dev
Δa	0.026	0.237	0.023	0.056
Δc	0.001	0.021	0.022	0.018
$R - R^f$	0.052	0.173	0.066	0.164
R^f	0.003	0.056	0.017	0.027

Table 12: Results: Consumption Growth Predictability

The Table shows coefficient estimates for cumulative consumption growth ($\Delta c_{t\to t+\tau}$) predictive regressions coming from model simulations (Panel A) and from post-war data (Panel B) using lagged advertising expenditures growth ($\Delta a_{t-1\to t}$) and consumption growth ($\Delta c_{t-1\to t}$) as predictors. The t-statistics in parentheses are computed using Hansen and Hodrick (1980) standard errors.

		Panel A: Model				Panel B: Data (Post-War)			
		$\Delta c_{t\to t+1}$	$\Delta c_{t\to t+2}$	$\Delta c_{t\to t+3}$	$\Delta c_{t\to t+4}$	$\Delta c_{t\to t+1}$	$\Delta c_{t\to t+2}$	$\Delta c_{t\to t+3}$	$\Delta c_{t\to t+4}$
(1)	$\Delta a_{t-1\to t}$	0.017	0.026	0.031	0.033	0.129	0.160	0.128	0.085
		(1.46)	(1.46)	(1.34)	(1.19)	(3.25)	(2.10)	(1.20)	(0.67)
	R^2_{adj}	0.052	0.045	0.036	0.030	0.143	0.074	0.018	-0.007
(2)	$\Delta c_{t-1\to t}$	0.220	0.313	0.329	0.292	0.266	0.183	0.043	-0.069
		(2.00)	(1.51)	(1.11)	(0.77)	(2.03)	(0.75)	(0.14)	(-0.19)
	R^2_{adj}	0.069	0.055	0.042	0.032	0.051	-0.006	-0.018	-0.018
(3)	$\Delta a_{t-1\to t}$	-0.001	0.007	0.020	0.036	0.127	0.201	0.195	0.161
		(-0.03)	(0.21)	(0.42)	(0.64)	(2.47)	(2.00)	(1.33)	(0.91)
	$\Delta c_{t-1\to t}$	0.228	0.249	0.147	-0.039	0.010	-0.214	-0.337	-0.380
		(1.16)	(0.57)	(0.21)	(-0.12)	(0.07)	(-0.71)	(-0.79)	(-0.74)
	R^2_{adj}	0.085	0.081	0.076	0.073	0.128	0.067	0.016	-0.010

Table 13: Results: Returns Predictability

The Table shows coefficient estimates for cumulative returns ($r_{t\to t+\tau}$) predictive regressions coming from model simulations (Panel A) and from post-war data (Panel B) using lagged advertising expenditures growth ($\Delta a_{t-1\to t}$) and consumption growth ($\Delta c_{t-1\to t}$) as predictors. The t-statistics in parentheses are computed using Hansen and Hodrick (1980) standard errors.

		Panel A: Model				Panel B: Data (Post-War)			
		$r_{t\to t+1}$	$r_{t\to t+2}$	$r_{t\to t+3}$	$r_{t\to t+4}$	$r_{t\to t+1}$	$r_{t\to t+2}$	$r_{t\to t+3}$	$r_{t\to t+4}$
(1)	$\Delta a_{t-1\to t}$	-0.090	-0.169	-0.238	-0.300	0.358	0.636	1.273	1.385
		(-1.37)	(-1.99)	(-1.90)	(-2.15)	(0.94)	(0.96)	(1.38)	(1.13)
	R^2_{adj}	0.029	0.037	0.041	0.047	-0.003	0.003	0.031	0.016
(2)	$\Delta c_{t-1\to t}$	-2.108	-3.875	-5.389	-6.760	-2.189	-3.410	-4.121	-6.843
		(-2.13)	(-2.74)	(-2.93)	(-3.12)	(-1.94)	(-1.81)	(-1.64)	(-2.12)
	R^2_{adj}	0.060	0.098	0.125	0.150	0.042	0.045	0.038	0.065
(3)	$\Delta a_{t-1\to t}$	0.245	0.439	0.604	0.755	1.332	2.204	3.492	4.644
		(1.45)	(1.59)	(1.74)	(1.90)	(3.24)	(3.24)	(3.57)	(3.51)
	$\Delta c_{t-1\to t}$	-4.402	-7.968	-10.997	-13.740	-4.712	-7.583	-10.731	-16.174
		(-2.22)	(-2.37)	(-2.58)	(-2.78)	(-3.66)	(-3.69)	(-3.82)	(-4.16)
	R^2_{adj}	0.100	0.156	0.196	0.227	0.150	0.175	0.246	0.277

Table 14: Predictability in the Centralized Economy

The Table shows coefficient estimates for cumulative consumption growth ($\Delta c_{t\to t+\tau}$) and returns ($r_{t\to t+\tau}$) predictive regressions coming from simulations of the centralized economy, using lagged advertising expenditures growth ($\Delta a_{t-1\to t}$) and consumption growth ($\Delta c_{t-1\to t}$) as predictors. The t-statistics in parentheses are computed using Hansen and Hodrick (1980) standard errors.

		Panel A: Consumption				Panel B: Returns			
		$\Delta c_{t\to t+1}$	$\Delta c_{t\to t+2}$	$\Delta c_{t\to t+3}$	$\Delta c_{t\to t+4}$	$r_{t\to t+1}$	$r_{t\to t+2}$	$r_{t\to t+3}$	$r_{t\to t+4}$
(1)	$\Delta a_{t-1\to t}$	-0.116	-0.228	-0.325	-0.378	-0.121	-0.213	-0.326	-0.403
		(-1.00)	(-1.51)	(-1.86)	(-1.90)	(-1.10)	(-1.41)	(-1.64)	(-1.82)
	R^2_{adj}	0.024	0.033	0.042	0.043	0.024	0.027	0.035	0.039
(2)	$\Delta c_{t-1\to t}$	-0.100	-0.197	-0.278	-0.327	-0.103	-0.184	-0.282	-0.350
		(-1.00)	(-1.80)	(-1.89)	(-2.00)	(-1.04)	(-1.30)	(-1.74)	(-1.92)
	R^2_{adj}	0.025	0.035	0.045	0.048	0.025	0.030	0.038	0.042
(3)	$\Delta a_{t-1\to t}$	0.159	0.343	0.385	0.552	0.113	0.293	0.398	0.597
		(0.24)	(0.42)	(0.53)	(0.44)	(0.15)	(0.32)	(0.45)	(0.43)
	$\Delta c_{t-1\to t}$	-0.219	-0.463	-0.581	-0.765	-0.186	-0.405	-0.588	-0.820
		(-0.40)	(-0.84)	(-0.87)	(-0.84)	(-0.32)	(-0.54)	(-0.69)	(-0.89)
	R^2_{adj}	0.041	0.047	0.054	0.058	0.041	0.041	0.050	0.054